647510 **MSC1MatC101x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Nov/Dec - 2022 [NEW COURSE]
Algebra 1(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (a) Answer any TWO of the following questions. **[10]**

(1) In standard notation, prove that Z(G) is a normal subgroup of a group G.

(2) Let H and K be two subgroups of a group G. Let $HK = \{hk | h \in H, k \in K\}$. Prove that HK is a subgroup of G if and only if $HK = KH$.

(3) Prove that every group of order p^2 is abelian, where p is a prime.

Q.1 (b) Answer any ONE of the following questions. **[04]**

(1) State and prove Cayley's theorem.

(2) State and prove First Isomorphism Theorem of groups.

Q.2 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Sylow's third theorem.

(2) Prove that every finite group is G is isomorphic to a permutation group (a subgroup of S_G).

(3) Prove that group of order 56 is not simple.

Q.2 (b) Answer any ONE of the following questions. **[04]**

(1) Let G be a finite abelian group and a prime p divides o(G). Let P be a sylow p-subgroup of G. Prove that P is the only sylow p-subgroup of G if and only if P is a normal subgroup of G.

(2) Define the following terms:

a). Sylow-*p*-Subgroup b). Internal direct product

Q.3 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Fundamental Theorem of Ring theory.

(2) State and prove Second Isomorphism Theorem of ring homomorphism.

(3) Prove that in a nonzero commutative ring with unity, an ideal M is maximal ideal if and only if R/M is a field.

Q.3 (b) Answer any ONE of the following questions. **[04]**

(1) Let G be an abelian group. Prove that $(ab)^n = a^n b^n$, for all $a, b \in G$ and for all $n \in N$.

(2) Define the following terms:

a). Ring b). Integral Domain c). Prime Ideal d). Maximal Ideal

Q.4 (a) Answer any TWO of the following questions.

(1) Let $f(x) \in \mathbb{Z}[x]$ be primitive polynomial. Then prove that $f(x)$ is reducible over $\mathbb{Q}$ if and only if $f(x)$ is reducible over $\mathbb{Z}$. **[10]**

(2) State and prove Unique Factorization Theorem.

(3) State and prove Division Algorithm Theorem.

Q.4 (b) Answer any ONE of the following questions. **[04]**

(1) Define Irreducible Polynomial and prove that $x^3 - 5x + 10$ is irreducible over $\mathbb{Q}$.

(2) Prove that product of two primitive polynomials is again a primitive polynomial.

Q.5 (a) Answer any TWO of the following questions. **[10]**

(1) State and prove Eisenstein Criterion.

(2) State and prove Third isomorphism theorem of groups.

(3) Define Subgroup and Normal Subgroup of a Group. Show that group of order 108 has a normal subgroup of order 27.

Q.5 (b) Answer any ONE of the following questions. **[04]**

(1) Define field with an example. Prove that a finite integral domain is a field.

(2) If R is a commutative ring with unity then prove that R is field if and only if R has precisely two ideals $\{0\}$ and R itself.

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